

Charm, Bottom evolution in DIS

- an attempt to generate $m \neq 0$ pdf's and describe F_2^c

$$F_2^c(x, Q^2) = e_c^2 \int_x^{z_{max}} dz \left\{ C_{g=c}(z) \frac{x}{z} c\left(\frac{x}{z}, Q^2\right) + C_g(z) \frac{x}{z} g\left(\frac{x}{z}, Q^2\right) \right\}$$

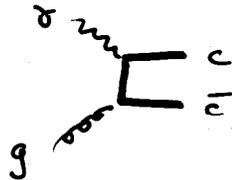
One approach

FFN $n_f=3$

$$c(x, Q^2) = 0$$

$$C_g(z, \frac{m_c^2}{Q^2}) = C^{PGF}(z, \frac{m_c^2}{Q^2})$$

eg. GRS



But (i) useful to generate a non-zero charm pdf for computing other processes.

(Should correspond to \overline{MS} $m=0$ generation of charm pdf in the $Q^2 \rightarrow \infty$ limit)

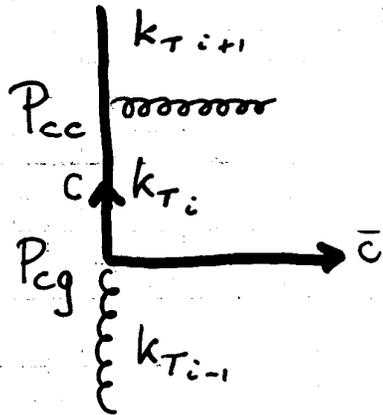
$$(ii) C^{PGF} \sim \alpha_s \ln \frac{Q^2}{m_c^2}$$

- these pot. large logs should be re-summed.

$$H.O. \text{ corrections also } \sim \left(\alpha_s \ln \frac{Q^2}{m_c^2} \right)^n$$

These are re-summed by DGLAP evolution.

Look at graphs involving charm in Leading Log Limit.



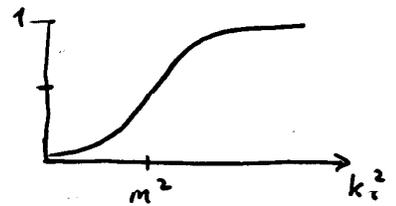
$$\sim \int \frac{dk_{T_{i-1}}^2}{k_{T_{i-1}}^2} \int \frac{dk_{T_i}^2 k_{T_i}^2}{(k_{T_i}^2 + m^2)^2} \int \frac{dk_{T_{i+1}}^2}{k_{T_{i+1}}^2} \dots$$

including m
here affects NNLO
only

strongly ordered $\dots \ll k_{T_{i-1}}^2 \ll k_{T_i}^2 \ll k_{T_{i+1}}^2 \ll \dots$

L.O. resums $(\alpha_s \ln Q^2)^n$

Approx $\frac{1}{1 + \frac{m^2}{k_{T_i}^2}} \approx \theta(k_{T_i}^2 - m^2)$



ie effect of $m \neq 0 \Rightarrow$ cut out $k_{T_i}^2 \lesssim m^2$

$k_{T_{i+1}}^2 \gg k_{T_i}^2$ implies neglect m^2 in $k_{T_{i+1}}$ integ.

\therefore At L.O.
evolution
eqns.

$$\dot{g} = P_{gg} \otimes g + \sum_q P_{gq} \otimes q + P_{gc} \otimes c$$

$$\dot{q} = P_{qq} \otimes q + P_{qg} \otimes g$$

$$\dot{c} = P_{cq} \otimes q + P_{cc} \otimes c$$

Thus at L.O. $P_{ci} = P_{ci} \Big|_{m=0} \theta(Q^2 - m^2)$

Mom. conserved by putting $P_{gg} \rightarrow P_{gg} - \frac{1}{3} \delta(1-z) * [3 + \theta(Q^2 - m^2)]$

At NLO, bit more complicated

but result is that $m \neq 0$ affect only $P_{cg}^{(0)}$

Look again at

$$\int \frac{dk_{T_i}^2 k_{T_i}^2}{(k_{T_i}^2 + m^2)^2}$$

$$= \int_{k_{T_{i-1}}^2}^{Q^2} \frac{d(k_{T_i}^2 + m^2)}{(k_{T_i}^2 + m^2)} - \int \frac{m^2 dk_{T_i}^2}{(k_{T_i}^2 + m^2)^2}$$

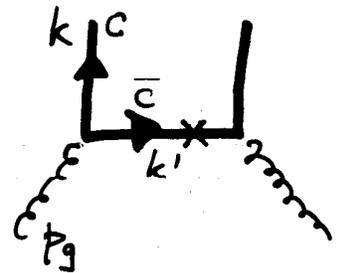
\downarrow usual $\ln \frac{Q^2}{m^2}$ for $k_{T_{i-1}}^2 \ll m^2$ equiv. to \mathcal{O} factor above

\downarrow const. contrib. \therefore NLO (ie α_s but no $\ln Q^2$)

\Rightarrow At NLO P_{cc} , P_{gc} remain as before

but $P_{cg} = P_{cg}^{(0)}(m \neq 0) + \alpha_s P_{cg}^{(1)}(m=0) \mathcal{O}(Q^2 - m^2)$

evaluate this using old-fash. PT dW_{cg}



$$dW_{cg} = \frac{\alpha_s}{2\pi} \frac{dz}{z} \frac{dQ^2}{Q^2} P_{cg}^{(0)}(z, m, Q^2)$$

Energy denom. $\Delta E = E_{c\bar{c}} - E_g = \frac{m^2 + k_T^2}{2z(1-z)p_g}$

Helps to determine the relevant scale Q^2

should have $\Delta t \sim \frac{1}{\Delta E} = \frac{2E_g}{Q^2}$
 $q \rightarrow c\bar{c}$

$$\Rightarrow Q^2 = \frac{m^2 + k_T^2}{z(1-z)} \geq \frac{m^2}{z(1-z)} \geq 4m^2$$

This is the 'natural' physical scale

- represents the value of Q^2 ($\geq 4m^2$)

when resolution is sufficient to observe

the individual c, \bar{c} in $g \leftrightarrow c\bar{c}$

fluctuation time Δt .

But when we consider $Q^2 \rightarrow \infty$, this scale does not correspond to \overline{MS} fact. scheme in $m=0$ limit!

$$Q^2 \rightarrow \bar{Q}^2 = Q^2 z(1-z) = m^2 + k_T^2 > m^2$$

and then

$$P_{cg}^{(0)} = \left\{ [z^2 + (1-z)^2] + \frac{2m^2}{Q^2} z(1-z) \right\} \theta(Q^2 - m^2)$$

Can now use the \overline{MS} $m=0$ expression for $P_{cg}^{(1)}$

$$P_{gg}^{(0)} \text{ adjusted} \rightarrow \Delta P_{gg}^{(0)} = -\frac{2}{3} \frac{1}{2} \delta(1-z) \frac{m^2}{2Q^2} \theta(Q^2 - m^2)$$

Note

If we go to small x , $W^2 \approx Q^2 \frac{1}{x}$

can be large (enough to produce charm)



but if $Q^2 \lesssim m^2$, then not enough to resolve $g \rightarrow c\bar{c}$ in proton

Coeff. fns For F_2^c when $m \neq 0$

$$F_2^c(x, Q^2) = 2e_c^2 \int_x^1 dz \frac{x}{z} \left\{ C_{g \rightarrow c}(z, Q^2, \mu^2) c\left(\frac{x}{z}, \mu^2\right) + C_g(z, Q^2, \mu^2) g\left(\frac{x}{z}, \mu^2\right) \right\}$$

$$C_c = C_c^{(0)} + \frac{\alpha_s}{4\pi} C_c^{(1)}$$

$$C_g = \frac{\alpha_s}{4\pi} C_g^{(1)}$$

For $Q^2 < m^2$ ($c(x, Q^2) = 0$) $C_g^{(1)} = \text{PGF}$
cross-section

As $Q^2 \rightarrow$ $C_c \otimes c$ increases rapidly & dominates.

Part of the Feynman diagram for $C_c \otimes c$
is contained in $C_g^{\text{PGF}} \otimes g$

- must subtract this, to avoid double counting

$$C_z^{(0)}(z, Q^2) = z \delta\left(z - \frac{1}{1 + \frac{4m^2}{Q^2}}\right) \left(1 + \frac{4m^2}{Q^2}\right)$$

↑
comes from
 $(x'p+q)^2 = m^2$

↑ comes from
 $\frac{\sigma_L}{\sigma_T} = \frac{4m^2}{Q^2}$

$$C_g^{(1)} = C_g^{\text{PGF}} - \Delta C_g$$

$$\Delta C_g(z, Q^2) \approx \int_{Q_{\min}^2}^{Q^2} d \ln Q'^2 P_{cg}(z, Q'^2)$$

$$\xrightarrow{m \rightarrow 0} [z^2 + (1-z)^2] \ln \frac{Q^2}{m^2} + 2z(1-z)$$

$$\text{while } C_g^{\text{PGF}} \xrightarrow{m \rightarrow 0} [z^2 + (1-z)^2] \ln \left[\frac{Q^2}{m^2} \cdot \frac{1-z}{z} \right] + 8z(1-z) - 1$$

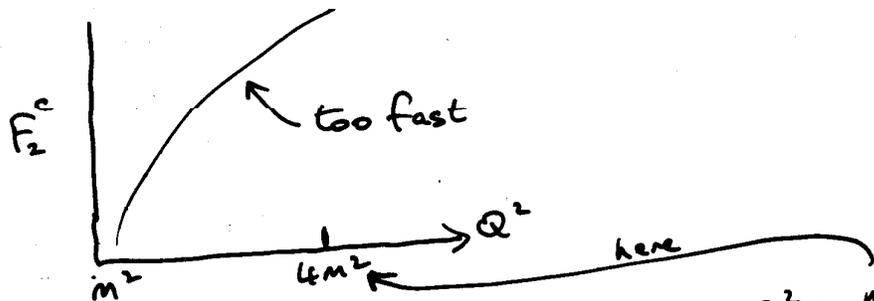
↑
correctly removed by
the subtraction to give
 $\sim C_g^{\overline{\text{MS}}}(z)$

So, in the $Q^2 \rightarrow \text{large}$ limit, retrieve evolution in $\overline{\text{MS}}$ scheme
At low Q^2 ($\sim m^2$) evolution will differ from ACOT VFNS
scheme by $O(\alpha_s^2)$
Olness WG1

$C_c^{(1)}$? — sufficient to take $m=0$ expression
since it contains no $\ln Q^2$ terms
(recall that with $m \neq 0$, we lose a log)

Confront data on $\overline{F_2^c}$

A problem: we switch on $c(x, Q^2)$ at $Q^2 = m^2$ now
(not $4m^2$)



Reason: physically reasonable scale was $Q^2 > \frac{m^2 + k_T^2}{z(1-z)}$

So, restore, by hand: introduce factor f into $C_c, \Delta C_g$

$$f = \left(1 - \frac{4m^2}{Q^2}\right) \theta\left(1 - \frac{4m^2}{Q^2}\right)$$

Boundary condition on α_s

- (i) $n_f = 4$ $\alpha_s(Q^2, 4)$ in terms of $\Lambda_{\overline{MS}}$
with β_0, β_1
using $n_f = 4$

(ii) Below m_c^2

$$\alpha_s^{-1}(3)(Q^2) = \alpha_s^{-1}(Q^2, 3) + \alpha_s^{-1}(m_c^2, 4) - \alpha_s^{-1}(m_c^2, 3)$$

At LO, this means the standard relation

$$\alpha_s(4)(Q^2) = \alpha_s(3)(Q^2) \left[1 + \frac{\alpha_s(3)}{6\pi} \ln \frac{Q^2}{m^2} \right]$$

Different components to F_2^c vs. Q^2 - Fig

At low Q^2 ($< 4m_c^2$) only PGF

At high Q^2 evolved $c(x, Q^2)$ dominates

Mismatch? - slight difference in scales between the two terms.

Compare with H1 and EMC data on F_2^c - Fig

- illustrates dependence on choice of m_c

1.2 \rightarrow 1.5

F_2^c/F_2 'large' in HERA range - Fig.

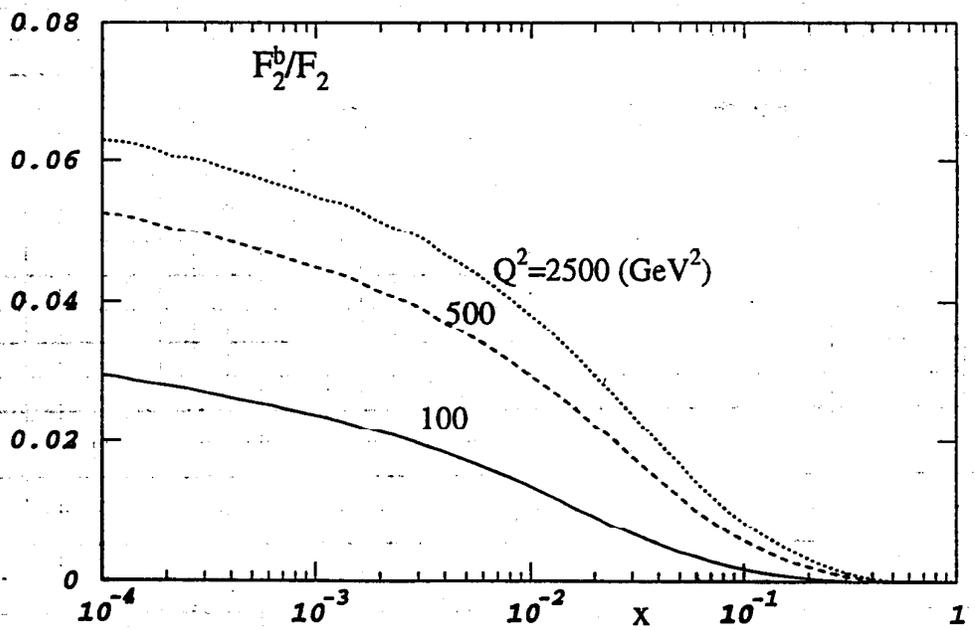
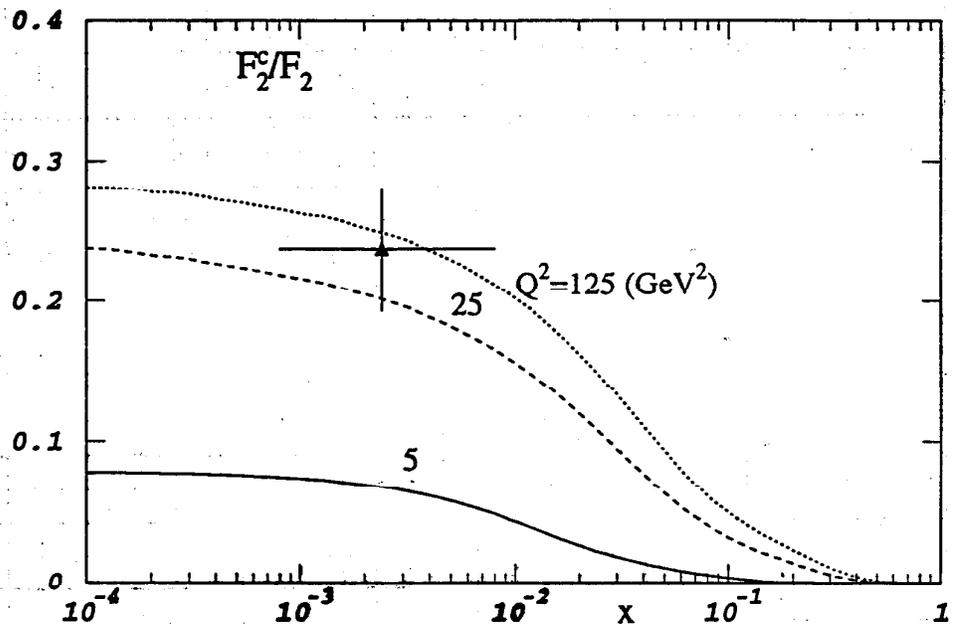


Fig. 8

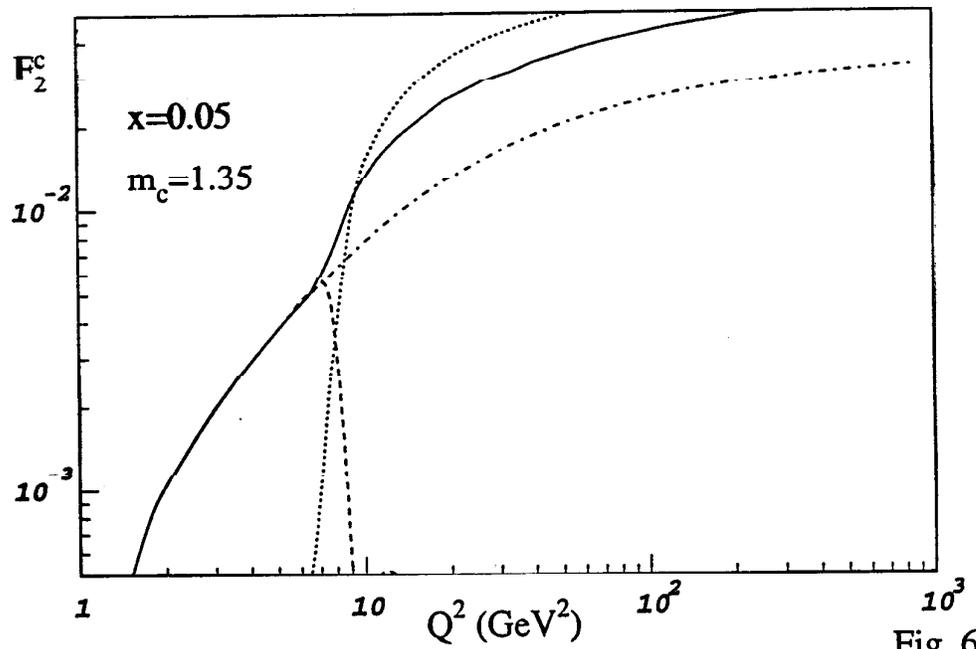
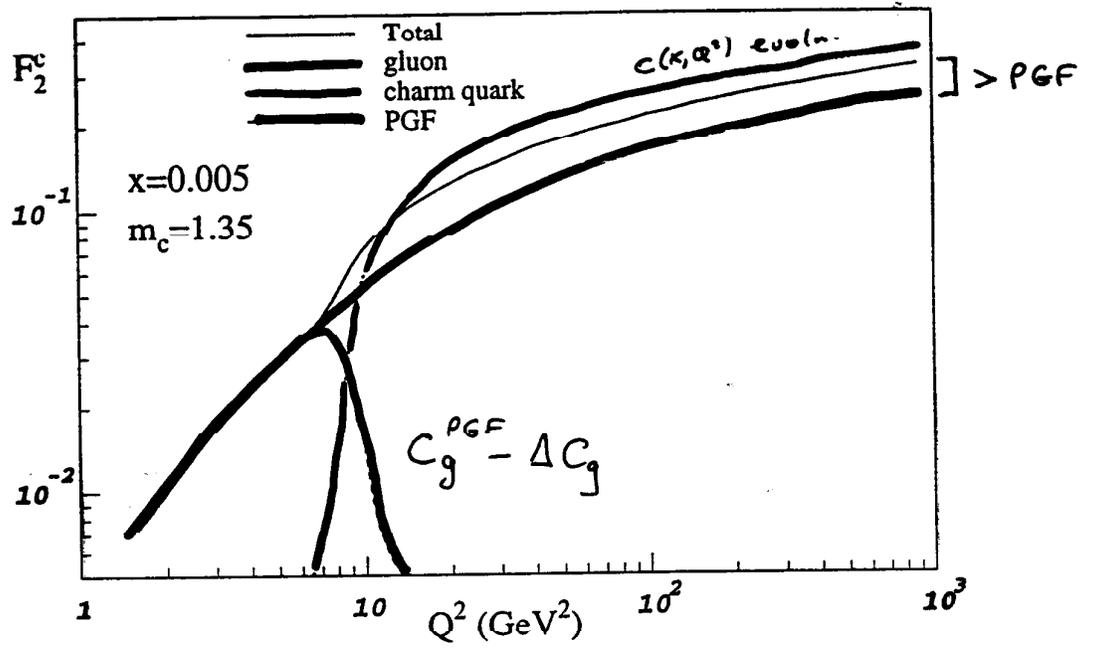


Fig. 6

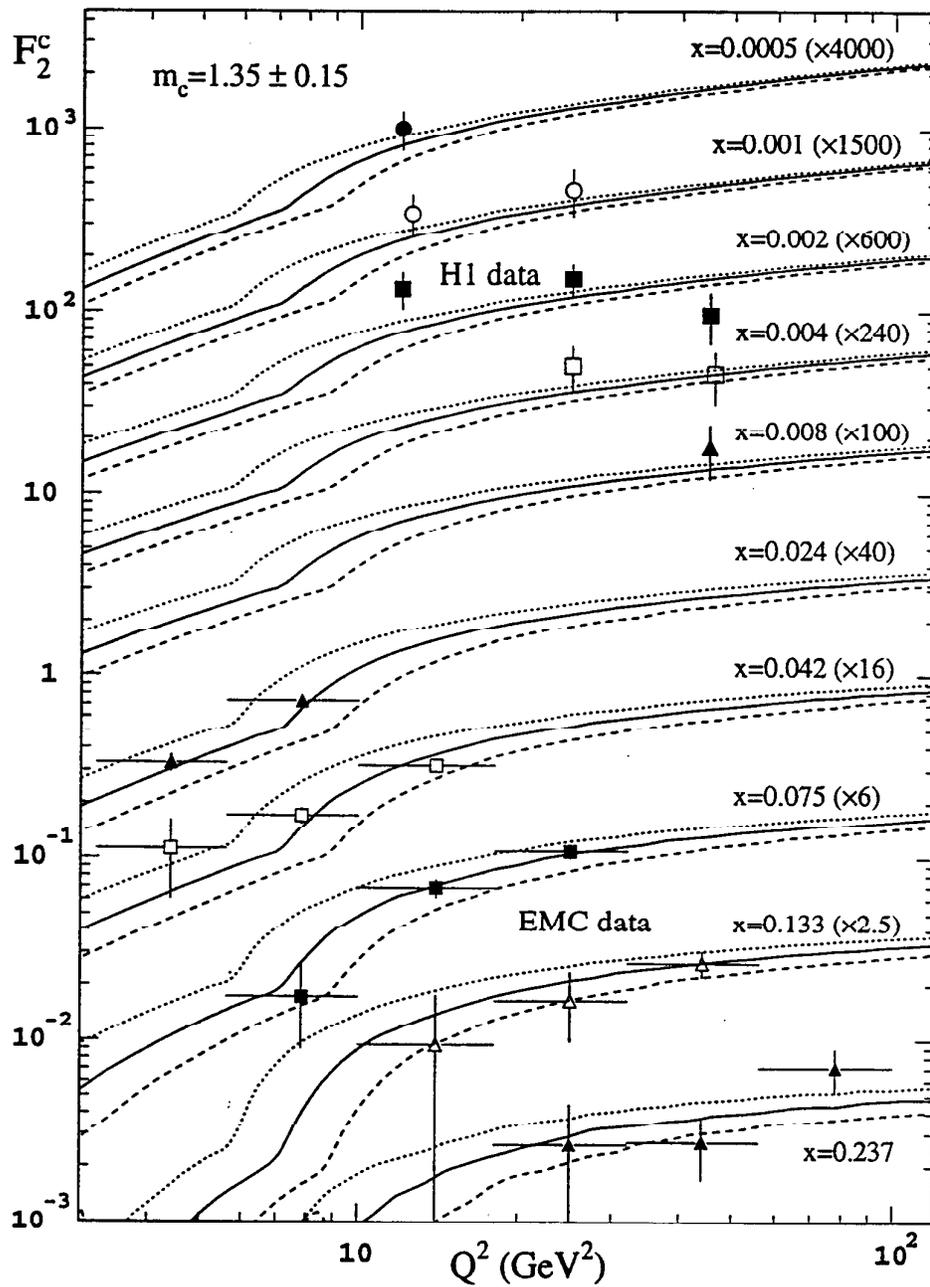


Fig. 7